

An Ionospheric Grid Algorithm for WAAS Based on the Minimum Mean Square Error Estimator

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ABSTRACT

The Wide-Area Augmentation System (WAAS) is a major component of the future air-navigation infrastructure of the United States. One of its functions is to provide corrections that can be used to reduce the errors inherent in the basic Global Positioning System (GPS) navigation solution available to civilian users. Among these are ionospheric delay corrections. WAAS also provides bounds on the post-correction residual errors from which the accuracy of the corrected navigation solution can be evaluated. The conservatism of the ionospheric estimation algorithm developed for WAAS Phase 1 is believed to limit the availability of the Precision Approach (PA) service delivered by the system more than necessary. MITRE's Center for Advanced Aviation System Development (CAASD) investigated an alternative algorithm based on the Minimum Mean Square Error (MMSE) technique. Preliminary performance results obtained from processing a limited amount of recorded GPS data with the "MMSE Algorithm" are presented in this paper. Given the amount and quality of these data, these initial results do not provide a good indication of expected performance under operational conditions. However, compared to the results of a Reference Algorithm somewhat similar to the WAAS Phase 1 algorithm, the MMSE Algorithm results suggest that a significant improvement in performance could be obtained from an operational system using this technique.

1. INTRODUCTION

The Wide Area Augmentation System (WAAS) is a major component of the future air-navigation infrastructure of the United States. It is currently under procurement by the Federal Aviation Administration (FAA). An initial implementation of WAAS, referred to as WAAS Phase 1, is scheduled to become operational in September 2000. According to its specification [1], the system should then offer an en route navigation to Nonprecision Approach (NPA) service with an availability of 0.999 over that portion of the entire service volume that is within the coverage of two geostationary communication satellites (Western States), and a limited Precision Approach (PA) service with an availability of 0.95 over 50 percent of the Continental United States (CONUS).

Future WAAS improvements will increase service availability for both en route and PA. While the schedule and the requirements for these improvements are now under review, current development efforts are underway to provide an en-route-to-NPA service with an availability of 0.99999 over the full service volume and a PA service with an availability of 0.999 over CONUS, Alaska, Hawaii, and Puerto Rico [1].

WAAS broadcasts information that allows the users of the system to improve the accuracy of their navigation solutions and compute bounds on that accuracy. This information includes vertical ionospheric delay estimates at

[◇] This paper is based on system analysis studies performed for the FAA GPS Product Team (AND-730). This paper reflects the views of the authors. Neither the Federal Aviation Administration nor the Department of Transportation makes any warranty or guarantee, or promise, expressed or implied, concerning the content or accuracy of the views expressed herein. This work was produced for the U.S. Government under Contract Number DTFA01-93-C-00001 and is subject to Federal Acquisition Regulation Clause 52.227-14, Rights in Data-General, Alt. III (JUN 1987) and Alt. IV (JUN 1987).

the nodes of an ionospheric grid and error bounds on those estimates.

The algorithm that was developed for WAAS Phase 1 to estimate the error bounds on the vertical ionospheric delay estimates, which is believed to be very conservative, does not appear capable of delivering the PA service availability that is required of WAAS Phases 2/3. Therefore, an effort aimed at developing a new algorithm capable of delivering a higher level of PA service availability, while preserving the same high level of integrity, was started.

MITRE/CAASD decided to investigate an algorithm based on the well-known Minimum Mean Square Error (MMSE) Estimator. The MMSE technique, which had been briefly looked at in the past [2], seemed to deserve further attention because of its formulation, which explicitly accounts for the correlations that exist among all the quantities of interest (unknowns as well as measurements).

2. BACKGROUND

WAAS is a Satellite-Based Augmentation System (SBAS). It broadcasts to its users various differential corrections via geostationary communication satellites (GEOs). Users apply these corrections to improve the accuracy of their measured pseudoranges to the Global Positioning System (GPS) satellites that they track, and thus, the accuracy of their position computations. These corrections are computed and formatted for uplink to the GEOs by WAAS Master Stations (WMSs), which process the pseudorange measurements made by WAAS Reference Stations (WRSs) scattered across the country.

Several sources of error affect the ranging information derived from measurements of the time-of-arrival at the receiver of signals broadcast by GPS satellites. They include: Selective Availability (SA), a technique by which the Department of Defense intentionally degrades the accuracy of the satellite ephemeris and/or clock information that is available to

civilian users; propagation delays caused by the ionosphere; propagation delays caused by the troposphere; and signal distortion caused by multipath propagation.

The propagation delay caused by the ionosphere along the line-of-sight between satellite and receiver can be measured directly, provided that the receiver can measure the time of arrival of signals broadcast by the satellite at two different frequencies. WRSs have this capability and perform both code and carrier-phase measurements on the two GPS frequencies known as L1 (1575.42 MHz) and L2 (1227.6 MHz). Most users, however, do not have this capability, and thus, need the corrections broadcast by WAAS.

While ionospheric delay is accumulated by the received signal as it crosses the altitudes between roughly 1,600 and 100 km, it was found convenient, and sufficiently accurate for navigation purposes, to model the received signal as being delayed by the ionosphere at one particular altitude. This approach is referred to as the *thin shell model*. In the case of WAAS, this imaginary ionospheric thin shell is located at a constant altitude of 350 km above the surface of the Earth. The point where the received signal crosses the thin shell is called an Ionospheric Pierce Point (IPP).

WAAS broadcasts messages containing ionospheric information for each node, or Ionospheric Grid Point (IGP), of an imaginary grid defined on the thin shell. IGPs are located at the intersection of lines of constant geodetic latitude and lines of constant longitude separated by 5 degrees (deg) between 55 deg latitude north and 55 deg latitude south¹ (wider spacings are used at higher latitudes). The information provided for each IGP consists of an estimated vertical delay (i.e., an estimated delay applicable to a signal crossing the thin shell perpendicularly) and a residual error bound called a Grid Ionospheric Vertical Error (GIVE).

¹ A proposal for extending the 5-deg spacing to 60 deg N and 60 deg S is currently under review by Working Group 2 of RTCA Special Committee SC-159.

A GIVE is essentially an estimate of the 99.9th percentile of the post-correction ionospheric delay error distribution for user IPPs in the neighborhood of the IGP.

User receivers compute a slant delay for each IPP using the grid vertical delays at the four IGPs surrounding the IPP (assuming they are all declared *monitored*) and then compute their positions from the corrected range measurements. They also compute error bounds on their computed positions from the GIVEs at the same IGPs, as well as other non-ionospheric error bounds. The resulting position bounds are called Horizontal Protection Level (HPL) and Vertical Protection Level (VPL); they are used to determine whether or not to proceed with certain flight operations such as, for example, a PA.

Formulas for carrying out these computations are given in [3]. Clearly, safety of flight requires that the HPL and VPL bound the residual errors in the user's computed position, called Vertical Position Error (VPE) and Horizontal Position Error (HPE), with a very high probability (0.9999999).

3. MMSE ALGORITHM DESCRIPTION

The ionospheric grid algorithm assessed in this report corresponds to a fairly straightforward application of the well known MMSE formulation. It computes both the vertical delay estimate and the GIVE at each IGP.

The design of the ionospheric grid algorithm presented in this paper (MMSE Algorithm hereafter) makes two key assumptions. First, it assumes that the required statistics of vertical delays (mean and variance) can be computed from the available set of ionospheric measurements, and that these statistics remain constant over the entire grid or, in other words, that they adequately characterize the vertical delay distribution at every point of the grid. Second, the design assumes that the correlation between the vertical delays at any two points is solely a function of the distance between the two points. Clearly, neither of these two

assumptions can be described as completely realistic; however, initial performance results indicate that the resulting algorithm design is sufficiently robust to tolerate such simplifications.

The MMSE Algorithm computes the vertical delays at the IGPs (\hat{x}) from the vertical delays (\underline{y}) derived from the slant delay measurements (*vertical delay measurements*) at the WRS IPPs as follows. First, the following four steps are performed at every epoch, i.e., every 10 seconds in the case of our simulation:

1. The mean value, μ_s , and variance, σ_s^2 , of all vertical delay measurements at the current epoch are computed. These statistics are then assumed to characterize all points in the grid, including all IGPs and all WRS IPPs, i.e., $\mu_x(i) = \mu_y(j) = \mu_s$ and $\sigma_x(i) = \sigma_y(j) = \sigma_s$, $\forall i, j$.

2. The initial covariance matrix is formed as follows:

$$[R_{xx}(i, j)] = \begin{cases} \sigma_x^2, & i = j \\ 0, & i \neq j \end{cases}$$

3. The covariance and cross-covariance matrices, R_{yy} and R_{xy} , are computed assuming an exponential decorrelation function with a constant decorrelation distance of 1,200 nmi. In other words, the elements of the R_{xy} and R_{yy} matrices are computed from the following formulas:

$$[R_{xy}(i, j)] = \sigma_x \sigma_y e^{-d(x_i, y_j)/D}$$

$$[R_{yy}(i, j)] = \sigma_y^2 e^{-d(y_i, y_j)/D}$$

where $D = 1,200$ nmi, $d(x_i, y_j)$ is the distance in nmi between IGP_{*i*} and IPP_{*j*}, and $d(y_i, y_j)$ is the distance in nmi between IPP_{*i*} and IPP_{*j*}.

4. The well-known MMSE formulas [4] are applied to compute the vector of vertical ionospheric delays at the IGPs, \hat{x} , and the corresponding vector of error variances, $\underline{\sigma_e}^2$.

$$\hat{\underline{x}} = R_{xy} R_{yy}^{-1} (\underline{y} - \underline{\mu}_y) + \underline{\mu}_x \quad (1)$$

$$\underline{\sigma}_e^2 = R_{xx} - R_{xy} R_{yy}^{-1} R_{yx} \quad (2)$$

A fifth step consisting of computing the GIVEs for all of the IGPs is executed once every user grid update cycle, i.e., every 5 minutes. First, the user grid is updated by setting the vertical delays at the IGP equal to the most recently computed vertical delays. Second, the GIVE (in meters) is computed at each IGP using the following formula:

$$GIVE_i = R_G \left(3.29 e^{d/D_{GIVE}} \sqrt{\max(\sigma_e^2)} \right) \quad (3)$$

where 3.29 is a constant factor intended to yield a GIVE that corresponds to the 99.9th percentile of the ionospheric delay post-correction errors, d is the distance in nmi between the IGP and the IPP closest to it, and D_{GIVE} is a constant decorrelation parameter equal to 900 nmi, the maximum σ_e^2 at IGP _{i} is taken over the 30 epochs preceding the user grid update. Finally, the function $R_G(\cdot)$ rounds the value of its argument up to the nearest value in the discrete set that can be accommodated by the message format specified in [3].

The computation of a GIVE is not constrained in any way by the number or location of neighboring WRS IPPs. The exponential factor in (3) is intended to capture the growth in the uncertainty associated with a GIVE estimate when the distance to the nearest IPP increases, thereby eliminating the need for a constraint on the number and location of neighboring IPPs. The only case in which an IGP can be declared *not monitored* is when the magnitude of its computed value happens to be larger than 45.0 m, the largest value that can be accommodated by the ionospheric message format specified in [3].

Remark. The ionospheric grid used for investigating the performance of the MMSE Algorithm extends over 30 deg in latitude and 40 deg in longitude. This choice was essentially

dictated by the available ground station data. The effect that grid size may have on the performance of the algorithm has therefore not been examined. However, it is quite possible that grid size does affect performance because, as stated above, the MMSE Algorithm implicitly assumes that the ionospheric conditions remain relatively constant over the entire grid. When this assumption is not quite satisfied, an averaging out of local differences in ionospheric activity takes place, which could result in a deterioration of performance.

This does not mean that the MMSE Algorithm cannot be used in conjunction with a large ionospheric grid such as the WAAS grid, but rather that a large grid may have to be subdivided into smaller (possibly overlapping) subgrids in order to obtain optimum performance from the algorithm. This is an implementation issue that will have to be examined in more detail as the algorithm undergoes further development and analysis.

4. REFERENCE GIVE ALGORITHM

Given the limitations of the available GPS data (five WRSs, no GEO data), the simulation results provide a pessimistic assessment of algorithm performance. Therefore, a *Reference GIVE Algorithm* similar to that developed for WAAS Phase I [5] is used to better assess whether the algorithm under investigation shows sufficient tangible benefits. No claim is made that the performance of the Reference GIVE Algorithm exactly matches that of the WAAS Phase 1 algorithm. However, the Reference GIVE Algorithm is believed to form a reasonable basis from which to assess the MMSE Algorithm.

The Reference GIVE Algorithm is a modified version of the algorithm described in [6] emulating that outlined in [5]. It can be briefly described as follows: For each IGP, the WRS IPPs located in the four grid cells surrounding the IGP on the last epoch of a (5-minute) user grid update cycle are counted. If at least three of the four cells contain at least one

IPP, the computation of the GIVE is allowed to proceed; otherwise the IGP is declared *not monitored*.

For each cell containing an IPP, a Root-Mean-Square (RMS) error is computed using all the IPPs accumulated in the cell since the last grid update (5 minutes). For this computation, error is defined as the difference between the measured vertical delay and the delay at the same location computed by bilinear interpolation from the grid information (using the formula specified in [3]). A 99.9 percent error bound is then obtained from the RMS error by multiplying it by a factor computed from the χ^2 distribution on the basis of the number of terms in the RMS computation. If all four cells surrounding the IGP contain at least one IPP trace, the largest 99.9 percent error bound is discarded. Finally, the GIVE is set equal to the largest remaining 99.9 percent error bound.

5. SIMULATION DESCRIPTION

The simulation used to assess the relative performance of the MMSE and Reference Algorithms processes data recorded by dual-frequency GPS receivers capable of making both code and carrier phase measurements. The simulation reads and processes the measurements and performs internal grid computations using the MMSE Algorithm at every epoch (10 seconds). Every user grid update cycle (5 minutes), the user grid is updated. Measurements from one WRS chosen to play the role of a user are then used to compute the user error bounds (VPL, HPL) and actual user errors (VPE, HPE), and collect results for the final performance metrics.

The input data set for the simulation consists of measurements recorded by five ground stations in April 1993 during a period of medium ionospheric activity, which included one severe geomagnetic storm (on April 4th). These sites are part of the FAA National Satellite Testbed (NSTB) and are located in

Atlantic City, NJ, Georgetown, SC, Dayton, OH, Boston, MA, and Oldtown, ME (Figure 1).



Figure 1. Simulated Ionospheric Grid

Pseudorange and carrier-phase measurements at L1 and L2 made by dual-frequency (Rogue) receivers for up to eight GPS satellites in view were recorded every 10 seconds (except at Boston where the receiver had only four channels). The data recorded on 15 different days, including April 4th when a severe geomagnetic storm swept the region (the 3-hour ap index reached 179), were used in the simulations. Combined, they amount to a total of 240 hours of data corresponding to a constellation that increased from 21 to 23 (on the last few days of the data) GPS satellites (not supplemented by any GEO ranging data).

Remark. The recorded GPS data available to this analysis results in a density of IPPs that is significantly lower than that expected in the WAAS Phase 1 (let alone WAAS Phases 2/3) operational environment. It follows that the simulation results obtained from these data are pessimistic and do not provide a good indication of the level of performance expected from WAAS Phase 1 (let alone WAAS Phases 2/3), particularly in the area of service availability. However, these results can be used to estimate relative performance improvements between

candidate algorithms, particularly near the boundaries of CONUS.

6. SIMULATION RESULTS

Availability and Integrity are well-defined concepts in the WAAS context.

Availability is the probability that the HPL and VPL computed by a user will be no greater than specified thresholds allowing the user to carry out a particular procedure such as, in the case of interest here, a PA. These thresholds, which are called Vertical Alert Limit (VAL) and Horizontal Alert Limit (HAL), are specified in the WAAS Specification [1].

Integrity is the probability that the HPL and VPL computed by a user will bound the actual residual errors in the user computed position (HPE, VPE).

HPL and VPL are computed from specified formulas that include terms accounting for all the main sources of error: clock, ephemeris, ionosphere, troposphere, and multipath [3]. In this analysis, the information necessary for computing the non-ionospheric error terms was not available. This was also true with respect to the computation of HPE and VPE. Thus, the following approach was taken. In order to compare VPL (HPL) to VAL (HAL), the non-ionospheric error terms were given standard values ($\sigma_{UDRE} = 0.6079$ m; $\sigma_{air} = 0.16 + 0.23 * \phi_{MP}$, where ϕ_{MP} is the multipath slant factor; and $\sigma_{tropo} = 0.12 * \phi_{tropo}$, where ϕ_{tropo} is the tropospheric slant factor [3]). On the other hand, to compare VPL (HPL) to VPE (HPE), the non-ionospheric error terms were set equal to zero (the subscript i is used in the tables below to indicate a quantity that includes the ionospheric error term only; the subscript a , a quantity that includes all error terms).

Because of these and other limitations (such as the incomplete GPS constellation and the lack of GEO ranging data), and to avoid possible confusion with availability results reported elsewhere, the results of this analysis are not presented in terms of availability and

integrity levels, but in terms of the percentages of the total number of user epochs for which $HPL \leq HAL$, $HPE \leq HPL$, $VPL \leq VAL$ and $VPE \leq VPL$.

In this analysis, only one user location is considered, Atlantic City, NJ. The values used for HAL and VAL are those in the current specification [1], i.e., $HAL = 30.0$ m and $VAL = 15.0$ m². Percentages are computed with respect to the number of user epochs during which signals were received from at least four satellites forming a “good” geometry, as characterized by a Vertical Dilution of Precision (VDOP) and a Horizontal Dilution of Precision (HDOP) simultaneously less than or equal to 3.5. (This value corresponds roughly to 99.9 percentile of VDOP availability in CONUS [7].)

The results obtained with the MMSE Algorithm are presented in Table 1. Those obtained with the Reference GIVE Algorithm are presented in Table 2.

Each table shows two columns of numbers: the first one, labeled *no storm*, corresponds to “average” ionospheric conditions ($A_p < 100$). (The input dataset excluded the data from the afternoon of April 4th.) The second one, labeled *storm*, corresponds to “severe” ionospheric conditions ($A_p > 100$). (The input dataset included only the data from the afternoon of April 4th.)

Table 1. MMSE Algorithm Results

	No Storm	Storm
Total User Epochs	67,967	2,284
Percent $VPE_i \leq VPL_i$	100.00	100.00
Percent $VPL_a \leq VAL$	96.66	35.29
Percent $HPE_i \leq HPL_i$	100.00	100.00
Percent $HPL_a \leq HAL$	100.00	100.00

² A proposal for separating between two services, CAT I PA with a VAL of 12 m and NPA with vertical guidance (NPV) with a VAL of 20 m is currently under review by the FAA.

Table 2. Reference Algorithm Results

	No Storm	Storm
Total User Epochs	67,967	2,284
Percent $VPE_i \leq VPL_i$	100.00	100.00
Percent $VPL_a \leq VAL$	9.05	0.66
Percent $HPE_i \leq HPL_i$	100.00	100.00
Percent $HPL_a \leq HAL$	23.28	25.88

While well below the target level of 99.9 percent, the percentages of epochs for which $VPL_a \leq VAL$ shown in Table 1 seem promising because they were obtained under unfavorable conditions resulting from a low density of WRS IPPs and a user location at the edge of coverage. As the results in Table 2 indicate, the MMSE Algorithm performed noticeably better than the Reference Algorithm under those extreme conditions. It is on this basis that the MMSE Algorithm appears promising even though the results of this initial evaluation do not provide a clear indication of the level of performance that could be expected from this algorithm in an operational WAAS environment.

It is also encouraging to note that the MMSE Algorithm appears to deliver a high degree of integrity. However, it should be emphasized that the results in Tables 1 and 2 were obtained with a dataset of about 240 hours of recorded GPS data, enough to possibly disqualify an inadequate algorithm, but clearly not enough to demonstrate the very high level of integrity required of an operational WAAS.

The main reason for the low percentages of epochs for which $VPL_a \leq VAL$ that were obtained with the Reference Algorithm is the requirement of the GIVE computation for at least one IPP in at least three of the four cells surrounding the IGP. Reducing this requirement to at least one IPP in at least one of the four cells surrounding the IGP improves the performance of the algorithm as shown in Table 3; however, not to the level of the MMSE Algorithm.

Table 3. Improvements From Reducing the IPP Density Requirement of the Reference GIVE Algorithm

	No Storm	Storm
Total User Epochs	67,967	2,284
Percent $VPE_i \leq VPL_i$	100.00	100.00
Percent $VPL_a \leq VAL$	81.24	21.72
Percent $HPE_i \leq HPL_i$	100.00	100.00
Percent $HPL_a \leq HAL$	96.95	93.43

The following figures show the distributions of GIVEs and VPLs that were obtained with the MMSE Algorithm. Note the nonlinear scale of the abscissa in Figure 2; the indicated set of GIVE values is that specified in [3]. Under “average” ionospheric conditions (no severe storm), the 50th GIVE percentile is 0.9 m and the 90th GIVE percentile is 1.8 m. Under severe storm conditions, the corresponding GIVE values are 2.7 m and 4.5 m.

Figure 3 shows that the estimation errors (VPE_i) are small. Under “average” ionospheric conditions 97.6 percent of them are below 2.0 m. Under severe storm conditions, 98.6 percent of them are below 8.0 m. Figure 3 also shows that, for both “average” and severe storm conditions, there is a fairly large margin between the VPE_i curve and the corresponding VPL_i curve. This suggests that further refinements to the algorithm might lead to greater performance improvements.

Figure 4 indicates that, when all sources of error are taken into account, the VPL_a is below 12 m 78.9 percent of the time during “average” ionospheric conditions, below 15 m 96.6 percent of the time, and below 20 m 99.6 percent of the time. While these numbers, once again, do not characterize the level of performance expected in an operational environment, they nevertheless suggest that the algorithm should be able to deliver a fairly high level of service availability even under unfavorable conditions.

7. CONCLUSIONS

An ionospheric grid estimation algorithm based on the MMSE technique was formulated and evaluated. Performance results were obtained for the MMSE and a Reference Algorithms using a computer simulation operating on a limited database of GPS data that were recorded at five ground stations in April 1993. The recorded data represent a variety of ionospheric conditions, including one severe storm. While the amount and quality of these data are inadequate to estimate the performance that could be expected from either of these two algorithms in an operational environment, the initial results obtained by processing these data are nevertheless encouraging. The significantly better performance obtained with the MMSE Algorithm as compared to the Reference Algorithm suggests that the MMSE technique may offer an avenue for developing an algorithm capable of delivering the high level of PA service availability required of WAAS without compromising the integrity of the system.

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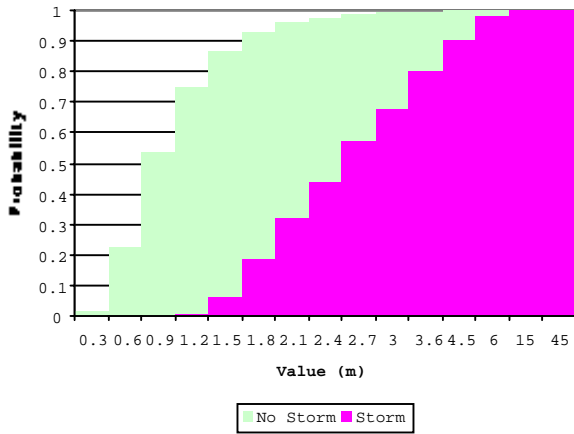


Figure 2. GIVE Distributions (CDFs)

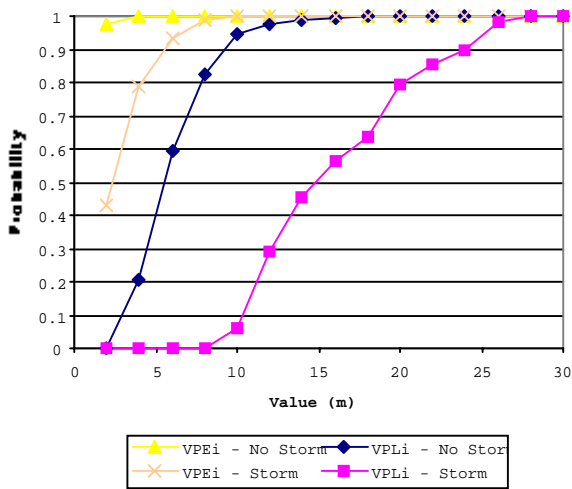


Figure 3. VPL_i/VPE_i Distributions (CDFs)

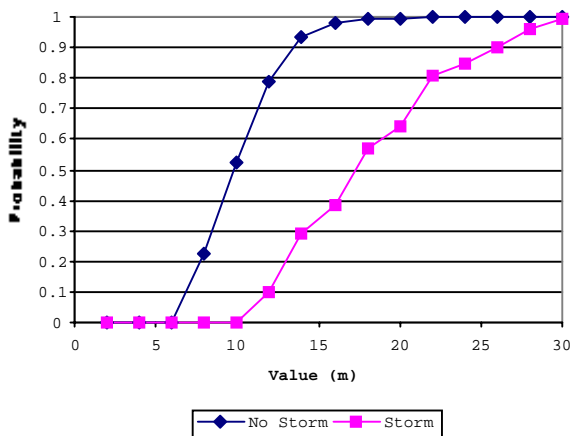


Figure 4. VPL_a Distributions (CDFs)